MATEMÁTICAS II
SOLUCIONES

OPCIÓN A

1. $(0,1,0) \in \pi$; $(3+\lambda, 5+\lambda, -1-\lambda) \in r$; $\vec{n} = (2,-1,2)$

$$d(P,\pi) = \frac{|\lambda|}{3} = 1 \Rightarrow \lambda = \pm 3 \Rightarrow \boxed{P_1(6,8,-4) \;;\; P_2(0,2,2)}$$

2. $\vec{v}_r = (1,1,2)$; $\vec{v}_s = (-1,-2,-1)$; $\vec{v}_r \times \vec{v}_s = (3,-1,-1)$

$$\boxed{\frac{x-2}{3} = \frac{y+1}{-1} = \frac{z-2}{-1}}$$

3. a)

$$M = \begin{pmatrix} 1 & k+1 & 2 \\ k & 1 & 1 \\ k-1 & -2 & -1 \end{pmatrix} \qquad \tilde{M} = \left(\begin{array}{ccc|c} 1 & k+1 & 2 & -1 \\ k & 1 & 1 & k \\ k-1 & -2 & -1 & k+1 \end{array}\right)$$

$$\det(M) = 2k^2 - 5k + 2 = 0 \Rightarrow k = 2,\ k = 1/2$$

$$\boxed{\begin{array}{l} k \notin \{\tfrac{1}{2}, 2\},\ rg(M) = 3,\ \text{S. Comp. Det.} \\ k = 2,\ rg(M) = rg(\tilde{M}) = 2,\ \text{S. Comp. Indet.} \\ k = \tfrac{1}{2},\ rg(M) = 2,\ rg(\tilde{M}) = 3,\ \text{S. Incompat.} \end{array}}$$

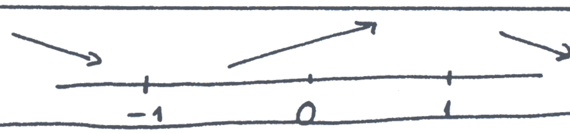
b)

$$k = 2 \Rightarrow \boxed{x = \frac{7}{5} - \frac{1}{5}\lambda,\ y = -\frac{4}{5} - \frac{3}{5}\lambda,\ z = \lambda}$$

4. a)

$$f'(x) = \frac{1-x^2}{(x^2+1)^2}$$

$$f''(x) = \frac{2x(x^2-3)}{(x^2+1)^3}$$



$$\boxed{\begin{array}{ll} \text{máx. relativo } x = 1 & \\ \text{mín. relativo } x = -1 & \end{array} \quad \text{P. Inflexión} \begin{cases} x = -\sqrt{3} \\ x = 0 \\ x = +\sqrt{3} \end{cases}}$$

b)

$$F(x) = \int \frac{3x^2+x+3}{x^2+1}dx = \int \left(3 + \frac{x}{x^2+1}\right)dx = 3x + \frac{1}{2}\ln(x^2+1) + C$$

$$F(0) = C = 4$$

$$\boxed{F(x) = 3x + \frac{1}{2}\ln(x^2+1) + 4}$$

OPCIÓN B

1. $B \cdot A = 2I \; ; \; A^2 = I$

$XA^2 + BA = A^2 \iff X + 2I = I \iff \boxed{X = -I = \begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -1 \end{pmatrix}}$

2. a) $\begin{cases} x + 2y - 3z = 3 \\ 2x + 3y + z = 5 \\ ax + y + bz = 1 \end{cases}$ ← debe ser combinación lineal de las anteriores

$\alpha(x + 2y - 3z - 3) + \beta(2x + 3y + z - 5) \equiv ax + y + bz - 1$

$\alpha + 2\beta = a \qquad -3\alpha + \beta = b$

$2\alpha + 3\beta = 1 \qquad -3\alpha - 5\beta = -1 \implies \alpha = 2, \beta = -1 \implies a = 0, \; b = -7$

$\boxed{y - 7z = 1}$

b) $\begin{cases} x + 2y - 3z = 3 \\ 2x + 3y + z = 5 \\ x + y + z = 4 \end{cases}$ $\boxed{x = \frac{25}{3}, \; y = -\frac{11}{3}, \; z = -\frac{2}{3}}$

3. $\vec{v}_s = (3,1,1) \quad P_s(5,0,-1)$

a) $\begin{vmatrix} x & y-1 & z-2 \\ 1 & -1 & 2 \\ 3 & 1 & 1 \end{vmatrix} = 0 \to \boxed{\pi: -3x + 5y + 4z - 13 = 0}$

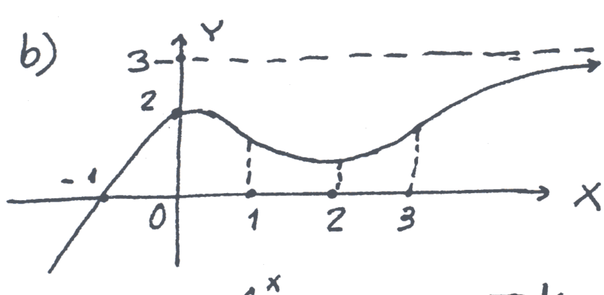
b) $d(s, \pi) = d(P_s, \pi) = \frac{|-15 - 4 - 13|}{\sqrt{50}} = \boxed{\frac{32}{\sqrt{50}} \, u}$

4. a) Asíntota horizontal: $y = 3$ para $x \to +\infty$

No hay asíntotas verticales.

Puede haber una asíntota oblícua para $x \to -\infty$

b)



c) $G(x) = \int_0^x g(t)\,dt \; ; \quad G'(x_0) = g(x_0) = 0 \iff \boxed{x_0 = -1}$